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Empirical Lepton–Quark Mass Formula: Four Dimensional State Space for Fermion Structure?

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It is observed that a simple mass formula of the form $m = \bar{m}Q^2(\exp \lambda)$ is wholly consistent with experimental measurements and quark model estimates for all 12 fundamental fermions. Here $\bar{m} = 433.3$ MeV is an input (mean fermion mass) constant, Q is the charge number of the lepton or quark, and λ is a real root of a quartic equation that brings in a principal quantum number n ($= 0, 1, 2, 3$). The charged lepton masses are given accurately to within 0.3 of 1%, all neutrino masses are zero, and the top mass is predicted by the formula to be close to $m_t = 163.6$ GeV.

1. INTRODUCTION

The purpose of this paper is to report a strikingly simple formula for the masses of all 12 leptons and quarks. With a postulated simple structural form that implicitly involves a principal quantum number n, the formula is manifestly predictive and yields mass values that may be accurate to within 0.3% for all fundamental fermions.

First a few words regarding motivation are in order. As is well known, the gauge bosons Z^0 and $W^{\pm}$ most probably derive their mass from interaction with a Higgs $SU(2)$ doublet. However, it appears to be less likely that leptons and quarks derive their mass primarily from a Higgs or other purely field-interaction mechanism. Indeed, attempts to account for lepton–quark mass by way of multi-Higgs extensions of the minimal standard model (e.g., Babu *et al.*, 1991) and alternative interaction mechanisms (Krauss *et al.*, 1993) lead to formidable difficulties and/or arbitrary

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coupling-parameter proliferation. Noteworthy also in interaction mass-generation schemes is the difficulty in achieving zero or near zero neutrino mass, as required by experiment and theory (Dolgov and Rothstein, 1993).

The neutrino mass problem "vanishes" quite literally if fundamental fermion mass is modulated by the square of the charge number, $m \propto Q^2$. Interestingly, the proportionality of fermion mass to charge-squared is a venerable notion, with perhaps the earliest antecedents in the classical $m_e = e^2/r_e$ electron self-energy constructs of J. J. Thomson and others (e.g., Born, 1957). In contemporary standard model theory, the quadratic form of gauge-invariant free-field electroweak energy supports the proportionality $m \propto Q^2$, but a structural model for the fundamental fermions is of course needed to internalize interactions.

2. CLASSIFICATION BASED ON LEPTON–QUARK PAIRS

Consider a mass formula of the form

$$m = \bar{m}Q^2(\exp \lambda) \tag{1}$$

where $\bar{m}$ is a mean fermion mass constant (an intermediate in the geometrical sense for all charged leptons and quarks) and λ is a dimensionless scaling parameter,[2] a quantity which depends on a fermion principal quantum number n. To express λ in terms of n, let us assume that an underlying structural commonality puts the 12 fermions into six lepton–quark pairs according to the arrangement shown in Table I. Note that the fermion pairs have definite values for the charge-baryonic projection operator

$$|Q - 2B| = \begin{cases} 0 & \text{for neutrinos and } |Q| = 2/3 \text{ quarks} \\ 1 & \text{for charged leptons and } |Q| = 1/3 \text{ quarks} \end{cases} \tag{2}$$

with either $|Q| = 0$, $2/3$ or $|Q| = 1$, $1/3$ for the lepton and quark in a pair. Also note that the lepton–quark pairs feature an ordering parity indicated by their positions inside the parentheses and the superscripts on the $|Q - 2B|$ values. By convention here, the ordering parity is even (odd) if the lepton (quark) appears to the left in the parentheses, and λ is negative (positive) for the lepton and positive (negative) for the quark in the pair. The ordering parity equals $(-1)^n$, i.e., is even (odd) if n is even (odd).

[2]If λ were to be related to a field-theoretic renormalization group, then the formula (1) suggests that the exact (all-order, diagonalized, and homogeneous) renormalization equation $\partial m/\partial \lambda = m$ is implied by the field theory for all lepton and quark masses.

Table I. Principal Quantum Numbers n, Associated Lepton–Quark Pairs, $|Q-2B|^{\pm}$ Values for Pairs, and λ Values from (5) and (6), as Shown in (7)

n	0		1	2		3
Pairs	(e, b)	(μ, s)	(d, τ)	(ν_e, t)	(ν_μ, c)	(u, ν_τ)
$\|Q-2B\|^{\pm}$	1^+		1^-	0^+		0^-
λ	$(-1-\sqrt{33}, -1+\sqrt{33})$	$(-\sqrt{2}, \sqrt{2})$	$(-\sqrt{2}, \sqrt{2})$	$(1-\sqrt{33}, 1+\sqrt{33})$	$(-2, 2)$	$(-2\sqrt{3}, 2\sqrt{3})$

Conversely, as indicated in Table I, the value of $|Q-2B|^{\pm}$ uniquely implies the value of n,

$$n = 2(1-|Q-2B|) + \tfrac{1}{2}[1-(\pm 1)] \tag{3}$$

where (± 1) is the ordering parity of the pair. The four values of n $(=0, 1, 2, 3)$ label the columns in Table I.

3. QUARTIC EQUATION FOR λ

With these classification preliminaries in place, the scaling parameter in (1) can be postulated to be a real root of the quartic equation

$$[\tfrac{1}{2}\lambda^2 - 16 + (i)^n\lambda](\tfrac{1}{2}\lambda^2 - n!) = 0 \tag{4}$$

Equation (4) is satisfied for real λ if

$$\tfrac{1}{2}\lambda^2 = 16 - (-1)^{n/2}\lambda \qquad \text{for} \quad n = 0, 2 \tag{5}$$

i.e., for n even, or if

$$\tfrac{1}{2}\lambda^2 = n! \qquad \text{for} \quad n = 0, 1, 2, 3 \tag{6}$$

i.e., for both even and odd n. The negative λ root of either quadratic equation (5) or (6) is associated with the left member of a lepton–quark pair in Table I, while the positive λ root of either (5) or (6) is associated with the right member of a lepton–quark pair. Hence, each fermion is identified completely by its $|Q-2B|^{\pm}$ state and whether its λ is "displaced-parabolic" and satisfies (5) or "quadratic-factorial" and satisfies (6). The λ's are

$$\begin{array}{llll}
\lambda_e = -1-\sqrt{33} & & \lambda_{\nu_e} = 1-\sqrt{33} & \\
\lambda_b = -1+\sqrt{33} & \lambda_d = -\sqrt{2} & \lambda_t = 1+\sqrt{33} & \lambda_u = -2\sqrt{3} \\
\lambda_\mu = -\sqrt{2} & \lambda_\tau = \sqrt{2} & \lambda_{\nu_\mu} = -2 & \lambda_{\nu_\tau} = 2\sqrt{3} \\
\lambda_s = \sqrt{2} & & \lambda_c = 2 &
\end{array} \tag{7}$$

Put into formula (1), the λ values in (7) (also shown in Table I) yield the mass values in Table II.

4. EMPIRICAL AGREEMENT

Table II shows the theoretical masses given by (1) and (7) for all leptons and quarks. The input mean fermion mass has been prescribed as $\bar{m} = 433.29$ MeV in order to make the geometrical mean of the theoretical muon and tau masses precisely equal to the experimental value (Aguilar-

Table II. Charge Numbers Q, Principal Quantum Numbers n, $|Q-2B|^{\pm}$ Values, and Theoretical Masses According to (1) and (7) with $\bar{m} = 433.29$ MeV as Prescribed Input

	Q	n	$\lvert Q-2B\rvert^{\pm}$	m (MeV)
e	-1	0	1^+	0.51010
μ	-1	0	1^+	105.34
s	$-1/3$	0	1^+	198.02
b	$-1/3$	0	1^+	5534.4
d	$-1/3$	1	1^-	11.704
τ	-1	1	1^-	1782.2
ν_e	0	2	0^+	0
ν_μ	0	2	0^+	0
c	2/3	2	0^+	1422.9
t	2/3	2	0^+	163.58×10^3
u	2/3	3	0^-	6.0278
ν_τ	0	3	0^-	0

Benitez *et al.*, 1992; Bai *et al.*, 1992), $\bar{m}^2 \equiv m_\mu m_\tau \equiv (m_\mu m_\tau)_{\text{exp}} =$ (105.658 MeV)(1776.9 MeV); the muon and tau masses are both within 0.3% of their respective experimental values,

$$-\delta m_\mu / m_\mu = \delta m_\tau / m_\tau = 3.0 \times 10^{-3} \tag{8}$$

The theoretical mass of the electron, $m_e = 0.51010$ MeV, is within 0.18% of its experimental value (Aguilar-Benitez *et al.*, 1992). Likewise, all quark masses given by (1) and (7) and displayed in Table II are in close agreement with potential model (pole mass) and/or running mass estimates (Dominguez and de Rafael, 1987; Gasser and Leutwyler, 1982; Donoghue and Holstein, 1992). In particular, the top mass is indicated to be close to $m_t = 163.6$ GeV.

Therefore formula (1) with the quartic equation (4) for λ gives experimentally admissible zero mass for the three neutrinos and accurately consistent mass values for the charged leptons and quarks over the five-order-of-magnitude range characterized by the ratio $m_t/m_e \cong 3.2 \times 10^5$. It is the simple dependence on n manifest in (4) that makes the formula patently predictive, with the 12 fermion masses constituting substantial output relative to the required (postulational) input.

5. PHYSICAL INTERPRETATION

It can be conjectured that formula (1) and the quantum numbers in Tables I and II arise in a structural model for the fundamental fermions:

formula (1) gives the mass operator of the structural model. Clues to the detailed nature of this structural model are provided by the forms (1)–(4). In particular, a natural interpretation of (4) is that of a characteristic equation satisfied by four-dimensional (4×4 matrix) operator λ with physical eigenvalues λ required to be real. To each admissible λ value shown in (7) and Table I, there is a four-dimensional eigenstate of the operator λ which relates the structural state of the associated lepton or quark for a prescribed value of $|Q-2B|^{\pm}$ and hence n [see equation (3)]. The physical fermion state or "structure" of a lepton or quark is thus associated with a direction from the origin in the four-dimensional state space, like a state of an internal spin-3/2 multiplet. However, the scaling operator λ has entries dependent on n (equivalently, on the projection operator-ordering parity $|Q-2B|^{\pm}$) which essentially effects the fermion mass symmetry-breaking $\bar{m} \to m$.

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